A Geometric Algorithm for Computing Zelevinsky Standard Representations

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(they/them)

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CMS Presentation

Roadmap

1. Notation for pKLH
2. Moduli spaces of Langlands parameters:
   Vogan varieties
3. The algorithm
4. Notable example
5. Future work

Notation

- $F/\mathbb{Q}_p$ denotes a p-adic field.
- $G = \mathbf{GL}_n(F)$ and $\widehat{G} = \mathbf{GL}_n(\mathbb{C})$.
- A langlands parameter $\phi : W_F \times \mathrm{SL}_2(\mathbb{C}) \to \widehat{G}$ is an admissible representation.
- $\lambda := \lambda_\phi : W_F \to \widehat{G}$ defined by

$$\lambda(w) = \phi\left(w, \mathrm{diag}\left(|w|^{1/2}, |w|^{-1/2}\right)\right)$$

is the **infinitesimal parameter** associated with ϕ.

[1] For more details see (Cunningham et al., 2022; Cunningham and Ray, 2023)

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Def^n: Cuspidal Support Category

$\mathrm{Rep}_\lambda(G)$ denotes the category of smooth representations of G with cuspidal support cut out by the Langlands correspondence applied to $\widehat{G}$-conjugacy classes of λ.[1]

[1] For more details see (Cunningham et al., 2022; Cunningham and Ray, 2023)

Standard Reps and pKLH

- For $\pi \in \text{Rep}_\lambda(G)$ irreducible, $\Delta(\pi)$ denotes the Zelevinsky standard representation with unique irreducible quotient π.[2]
- We are interested in determining the multiplicities of irreducibles in Jordan Hölder series of $\Delta(\pi)$

$$J(\Delta(\pi)) = \{m(\pi';\pi)\pi' : \pi' \in \text{Rep}_\lambda(G)^{irr}, m(\pi';\pi) = \text{multiplicity}\}$$

[2] For more details see (Bernstein and Zelevinsky, 1977; Zelevinsky, 1980)

[3] For more details see (Achar, 2021; Cunningham and Ray, 2023)

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Idea behind pKLH

If $m := (m_{i,j})_{i,j\in I}$ is the multiplicity matrix for standard representations in $\text{Rep}_\lambda(G)$, then $m = {}^t m^{geo}$ for m^{geo} a matrix of simple perverse sheaf stalk dimensions.[3]

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Geometry: Moduli Space of Langlands Parameters

Vogan Variety

The Vogan Variety associated with λ is defined as

$$V_\lambda := \{M \in \text{Lie } Z_{\widehat{G}}(\lambda(I_F)) : \lambda(\mathfrak{fr})M\lambda(\mathfrak{fr})^{-1} = q_F M\}$$

where $q_F = |k(\mathcal{O}_F)|$ and $I_F \leq W_F$ is the inertia subgroup.[4]

[1]Further details in (Cunningham et al., 2022, Sec. 4.2)

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Rmk:

In the case of $G = \mathrm{GL}_n(F)$, an infinitesimal parameter λ is characterized by the image of frobenius which is a semi-simple element of the form

$$\lambda(\mathfrak{fr}) = \text{diag}(q_F^{e_0}, ..., q_F^{e_{n-1}})$$

for $e_0 \geq \cdots \geq e_{n-1} \in \frac{1}{2}\mathbb{Z}$ and a specific choice of basis.

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Conjugation Action and Toy Example

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Toy Example: Take $G(F) = \mathbf{GL}_2(F)$, $\widehat{G} = \mathbf{GL}_2(\mathbb{C})$, and $\lambda(\mathfrak{fr}) = \mathrm{diag}\left(q_F^{1/2}, q_F^{-1/2}\right)$. Then

$$V_\lambda = \left\{ \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix} \in M_{2,2}(\mathbb{C}) : x \in \mathbb{C} \right\}, \quad H_\lambda = \left\{ \begin{pmatrix} t_1 & 0 \\ 0 & t_2 \end{pmatrix} \in \mathbf{GL}_2(\mathbb{C}) : t_1, t_2 \in \mathbb{C}^\times \right\}$$

V_λ has two orbits under the action by H_λ:

$$C_0 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \right\}, \; C_1 = \left\{ \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix} \in M_{2,2}(\mathbb{C}) : x \in \mathbb{C}^\times \right\}$$

Perverse sheaves

- $\mathrm{Per}_{H_\lambda}(V_\lambda)$ denotes the category of equivariant perverse sheaves on V_λ.[5]

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The simple objects of $\mathrm{Per}_{H_\lambda}(V_\lambda)$ for $G = \mathrm{GL}_n(F)$ are of the form

$$\{\mathcal{IC}(\mathbb{1}_C) : C \in \mathrm{Orbs}_{H_\lambda}(V_\lambda)\} \tag{1}$$

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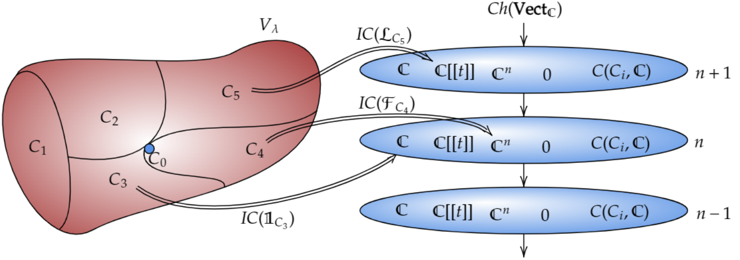

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The Algorithm: Goal

- For $C, C' \in \mathsf{Orbs}_{H_\lambda}(V_\lambda)$,

$$m^{geo}_{C,C'} = (-1)^{\dim C} \sum_{n \in \mathbb{Z}} (-1)^n \dim \mathcal{H}^n(\mathcal{IC}(\mathbb{1}_C))_{x_{C'}}$$

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New/Equivalent Goal

Determine the chain of vector spaces $\mathcal{H}^n(\mathcal{IC}(\mathbb{1}_C))_{x_{C'}}$ for all H_λ-orbits C, C' of V_λ.

The Algorithm: First Induction

Rmk: Partial Ordering on Orbits

We define a partial ordering on H_λ-orbits in V_λ by

$$C \leq C' \iff C \subseteq \overline{C'}$$

1. Find and store orbits using Zelevinsky multisegment machinery.[6]
2. Proceed inductively up the poset tree layer by layer.

[6] For further details see (Zelevinskii, 1981; Bernstein and Zelevinsky, 1977; Zelevinsky, 1980)

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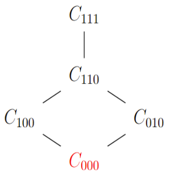

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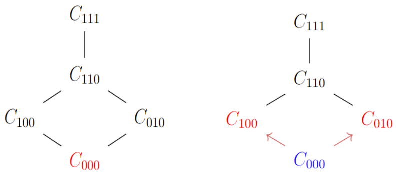

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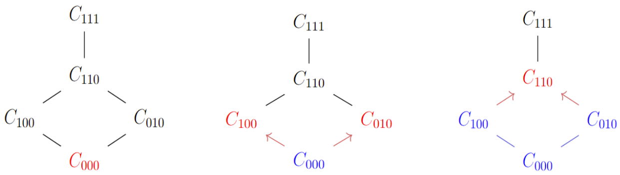

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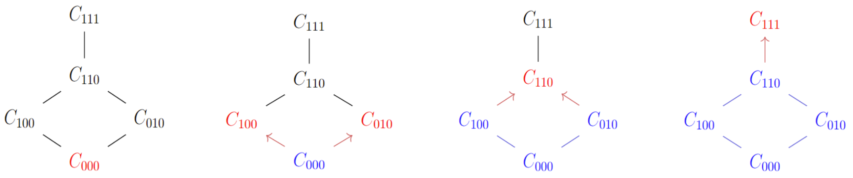

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The Algorithm: Smooth Closures

General Result[7]

If C, C' are H_λ orbits of V_λ, then

1. $\mathcal{IC}(\mathbb{1}_C)_{x_C} = \mathbb{C}[\dim C]$ for $x_C \in C$
2. $\mathcal{IC}(\mathbb{1}_C)_{x_{C'}} = 0$ for $x_{C'} \in C'$ if $C' \not\leq C$

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Smooth Closure[8]

If C, C' are H_λ orbits of V_λ with $\overline{C}$ smooth and $C' \leq C$, then

$$\mathcal{IC}(\mathbb{1}_C)_{x_{C'}} = \mathbb{C}[\dim C]$$

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The Algorithm: Singular Closures

- We wish to find a smooth space $\widetilde{C}$ with a proper birational map

$$\pi : \widetilde{C} \to \overline{C}$$

- This problem has a known solution in the case of H_λ orbits in V_λ for $G = \mathsf{GL}_n$.[9]

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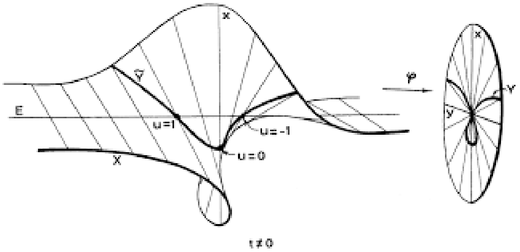

Figure: Resolution of Singularities through blow-up (Hatcher, Algebraic Geometry)

[9] For further details see (Abeasis, Del Fra, and Kraft, 1981; Reineke, 2001)

The Algorithm: Decomposition Theorem

Decomposition Theorem

If C is an H_λ orbit in V_λ and $\pi : \widetilde{C} \to \overline{C}$ is a resolution of singularities, then

$$R\pi_! \mathcal{IC}(\mathbb{1}_{\widetilde{C}_{sm}}) \cong \bigoplus_{i=-r(\pi)}^{r(\pi)} {}^{\mathfrak{p}}\mathcal{H}^i(R\pi_! \mathcal{IC}(\mathbb{1}_{\widetilde{C}_{sm}}))[-i]$$

$$\cong \bigoplus_{i=-r(\pi)}^{r(\pi)} \bigoplus_{C' \leq C} m_i(C'; C)\mathcal{IC}(\mathbb{1}_{C'})[-i]$$

where $r(\pi) = \max_{C' < C}(\dim C' + 2\dim \pi^{-1}(\{x_{C'}\}) - \dim C)$.[10]

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- For $x \in \overline{C}$,

$$(R\pi_! \mathcal{IC}(\mathbb{1}_{\widetilde{C}_{sm}}))_x \cong H^\bullet(\pi^{-1}(\{x\}))[\dim \widetilde{C}]$$[11]

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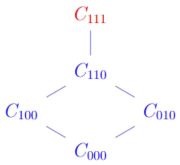
The Algorithm: Second Induction



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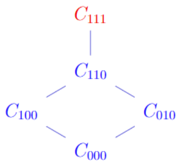

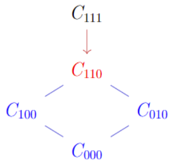

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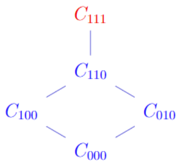

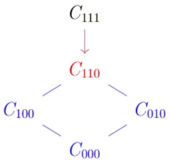

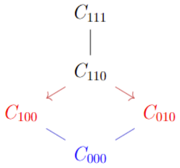

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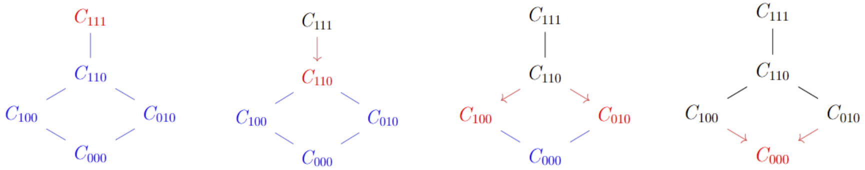

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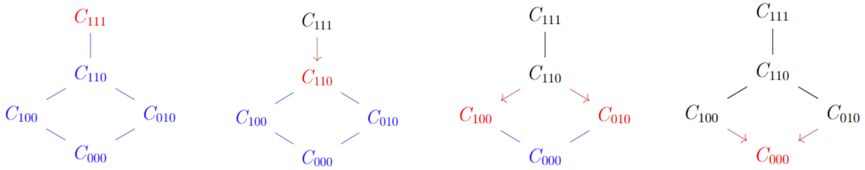

- Restricting to C',

$$\mathcal{IC}(\mathbb{1}_C)_{x_{C'}} \oplus \bigoplus_{i=-r(\pi)}^{r(\pi)} m_i(C';C)\mathcal{IC}(\mathbb{1}_{C'})_{x_{C'}}[-i]$$

$$\cong \mathcal{IC}(\mathbb{1}_C)_{x_{C'}} \oplus \bigoplus_{i=-r(\pi)}^{r(\pi)} m_i(C';C)\mathbb{C}[\dim C' - i]$$

equals $H^\bullet(\pi^{-1}(\{x_{C'}\}))[\dim C]$ after removing occurrences of $m_i(C'';C)\mathcal{IC}(\mathbb{1}_{C''})_{x_{C'}}[-i]$ for $C' < C'' < C$.

The Algorithm: Multiplicity

- We set $m_i(C';C) =$ the dimension of the vector space shifted by $\dim C' - i$, for $i \in [0, r(\pi)]$.
- By Poincaré-Verdier Duality we can then determine $m_{-i}(C';C) = m_i(C';C)$ for each i.[12]

[12] For further details see (Cataldo and Migliorini, 2007)

[13] For further details see (Achar, 2021, Lem 3.3.11)

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Rmk: Support

The non-trivial vector spaces for $\mathcal{H}^n(\mathcal{IC}(\mathbb{1}_C))_{x_{C'}}$ are located in degrees n (or shifts $-n$) such that $\dim C' \leq -n \leq \dim C$, with equality $-n = \dim C'$ if and only if $C' = C$. [13]

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Known Cases

Question:

Can we use this to determine all IC's yet?

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I1. $\lambda(\mathfrak{fr}) = \text{diag}\left(q_F^{(n-1)/2}, q_F^{(n-3)/2}, \ldots, q_F^{-(n-1)/2}\right)$, $V_\lambda \cong \mathbb{C}^n$

I2. $\lambda(\mathfrak{fr}) = \text{diag}\left(\underbrace{q_F^{1/2}, \ldots, q_F^{1/2}}_{\ell}, \underbrace{q_F^{-1/2}, \ldots, q_F^{-1/2}}_{k}\right)$, $V_\lambda \cong M_{\ell,k}(\mathbb{C})$

I3. $\lambda(\mathfrak{fr}) = \text{diag}\left(q_F^{1}, \underbrace{q_F^{0}, \ldots, q_F^{0}}_{\ell}, \underbrace{q_F^{-1}, \ldots, q_F^{-1}}_{k}\right)$, $V_\lambda \cong M_{1,\ell}(\mathbb{C}) \times M_{\ell,k}(\mathbb{C})$

Example of a Known Case

In the case of $G = \mathrm{GL}_5(F)$, with $\lambda(\mathfrak{fr}) = \mathrm{diag}(q_F^1, q_F^0, q_F^0, q_F^{-1}, q_F^{-1})$,

m_{geo}^{λ}	C_{000}	C_{010}	C_{100}	C_{110}	C_{111}	C_{200}	C_{211}
$\mathcal{IC}(\mathbb{1}_{C_{000}})$	$\mathbb{C}[0]$	0	0	0	0	0	0
$\mathcal{IC}(\mathbb{1}_{C_{010}})$	$\mathbb{C}[2]$	$\mathbb{C}[2]$	0	0	0	0	0
$\mathcal{IC}(\mathbb{1}_{C_{100}})$	$\mathbb{C}[3] \oplus \mathbb{C}[1]$	0	$\mathbb{C}[3]$	0	0	0	0
$\mathcal{IC}(\mathbb{1}_{C_{110}})$	$\mathbb{C}[4] \oplus \mathbb{C}[2]$	$\mathbb{C}[4]$	$\mathbb{C}[4]$	$\mathbb{C}[4]$	0	0	0
$\mathcal{IC}(\mathbb{1}_{C_{111}})$	$\mathbb{C}[5] \oplus \mathbb{C}[3]$	$\mathbb{C}[5] \oplus \mathbb{C}[3]$	$\mathbb{C}[5]$	$\mathbb{C}[5]$	$\mathbb{C}[5]$	0	0
$\mathcal{IC}(\mathbb{1}_{C_{200}})$	$\mathbb{C}[4]$	0	$\mathbb{C}[4]$	0	0	$\mathbb{C}[4]$	0
$\mathcal{IC}(\mathbb{1}_{C_{211}})$	$\mathbb{C}[6]$	$\mathbb{C}[6]$	$\mathbb{C}[6]$	$\mathbb{C}[6]$	$\mathbb{C}[6]$	$\mathbb{C}[6]$	$\mathbb{C}[6]$

Example of a Known Case

In the case of $G = \mathrm{GL}_5(F)$, with $\lambda(\mathfrak{fr}) = \mathrm{diag}(q_F^1, q_F^0, q_F^0, q_F^{-1}, q_F^{-1})$,

m_{geo}^{λ}	$\vert_{C_{000}}$	$\vert_{C_{010}}$	$\vert_{C_{100}}$	$\vert_{C_{110}}$	$\vert_{C_{111}}$	$\vert_{C_{200}}$	$\vert_{C_{211}}$
$\mathcal{IC}(\mathbb{1}_{C_{000}})$	$\mathbb{C}[0]$	0	0	0	0	0	0
$\mathcal{IC}(\mathbb{1}_{C_{010}})$	$\mathbb{C}[2]$	$\mathbb{C}[2]$	0	0	0	0	0
$\mathcal{IC}(\mathbb{1}_{C_{100}})$	$\mathbb{C}[3] \oplus \mathbb{C}[1]$	0	$\mathbb{C}[3]$	0	0	0	0
$\mathcal{IC}(\mathbb{1}_{C_{110}})$	$\mathbb{C}[4] \oplus \mathbb{C}[2]$	$\mathbb{C}[4]$	$\mathbb{C}[4]$	$\mathbb{C}[4]$	0	0	0
$\mathcal{IC}(\mathbb{1}_{C_{111}})$	$\mathbb{C}[5] \oplus \mathbb{C}[3]$	$\mathbb{C}[5] \oplus \mathbb{C}[3]$	$\mathbb{C}[5]$	$\mathbb{C}[5]$	$\mathbb{C}[5]$	0	0
$\mathcal{IC}(\mathbb{1}_{C_{200}})$	$\mathbb{C}[4]$	0	$\mathbb{C}[4]$	0	0	$\mathbb{C}[4]$	0
$\mathcal{IC}(\mathbb{1}_{C_{211}})$	$\mathbb{C}[6]$	$\mathbb{C}[6]$	$\mathbb{C}[6]$	$\mathbb{C}[6]$	$\mathbb{C}[6]$	$\mathbb{C}[6]$	$\mathbb{C}[6]$

- In $\mathrm{Rep}_\lambda(\mathrm{GL}_5(F))$ C_{111} and C_{211} correspond to standard representations
  $\Delta_{a_{111}} = I_{P_{1,3,1}}^{\mathrm{GL}_5}\left(\nu^0 \boxtimes Q\left(I_B^{\mathrm{GL}_3}\left(\nu^{-1} \boxtimes \nu^0 \boxtimes \nu^1\right)\right) \boxtimes \nu^{-1}\right)$ and
  $\Delta_{a_{211}} = I_{P_{3,2}}^{\mathrm{GL}_5}\left(Q\left(I_B^{\mathrm{GL}_3}\left(\nu^{-1} \boxtimes \nu^0 \boxtimes \nu^1\right)\right) \boxtimes Q\left(I_B^{\mathrm{GL}_2}\left(\nu^{-1} \boxtimes \nu^0\right)\right)\right)$
  where $\nu = |\det \cdot|_F$, and the table tells us that

$$J(\Delta_{a_{111}}) = \{Q(\Delta_{a_{111}}), Q(\Delta_{a_{211}})\}$$

Conclusions and Future Work

Results:

1. Able to compute intersection cohomology complexes making up the simple perverse sheaves for a Vogan variety attached to GL_n.
2. Using the pKLH this result can be transferred back to the decompositions of standard representations of $\mathrm{GL}_n(F)$.

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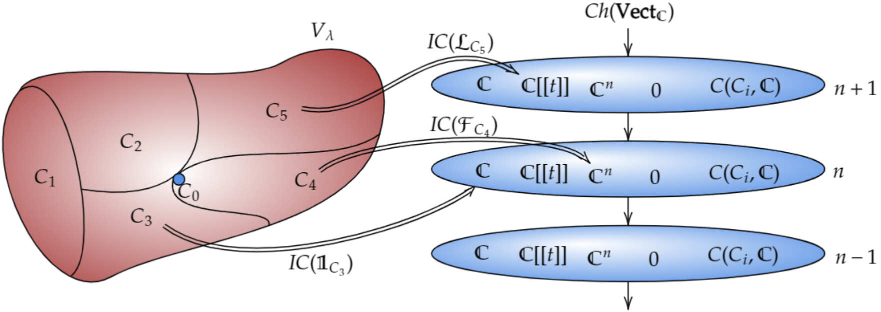
Questions?

Thank you for your time!

Any questions?



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